Physics Lecture 21 - Speed Loss from Wheel Bumps

Introduction

If you go to the website forum called "PWD Racing", I have been honored to moderate a forum topic there called "Pinewood Derby Physics". Some recent discussions on this topic may be found (need to log into the forum first) at the website:http://pwdracing.proboards91.com/ website. Here a question is posed, namely:

"Is it better to have added weight laterally distributed or tightly concentrated along the centerline of the car? One camp argues lateral distribution provides better 'stability'. Opposite camp argues increased lateral distribution increases roll inertia and costs time when wheels ride up and over track debris."

The stability argument is one that doesn't apply here—at least under the assumption that the PWD bodies are perfectly rigid and not flexible. The force on the individual axle/wheel bore surfaces may vary with mass distribution but the overall effect on speed is independent of the distribution of mass within the confines of the rigid body (as shown in **Lecture 11**). Also, wheel alignment accuracy and "rail riding" techniques do not necessarily depend on main body width if the wheel struts from a narrow body are fabricated properly. Only air drag will change with frontal area.

But the second part of the question is something that needs to be investigated and can indeed be analyzed with physics. The analysis is straightforward. We will deal with 4 situations:

Case 1: Off center wheel - (**a**) Wide body and (**b**) Narrow body
Case 2: Sharp bump on track - (**a**) Wide body and (**b**) Narrow body

Fundamentals of Rigid Body Motion

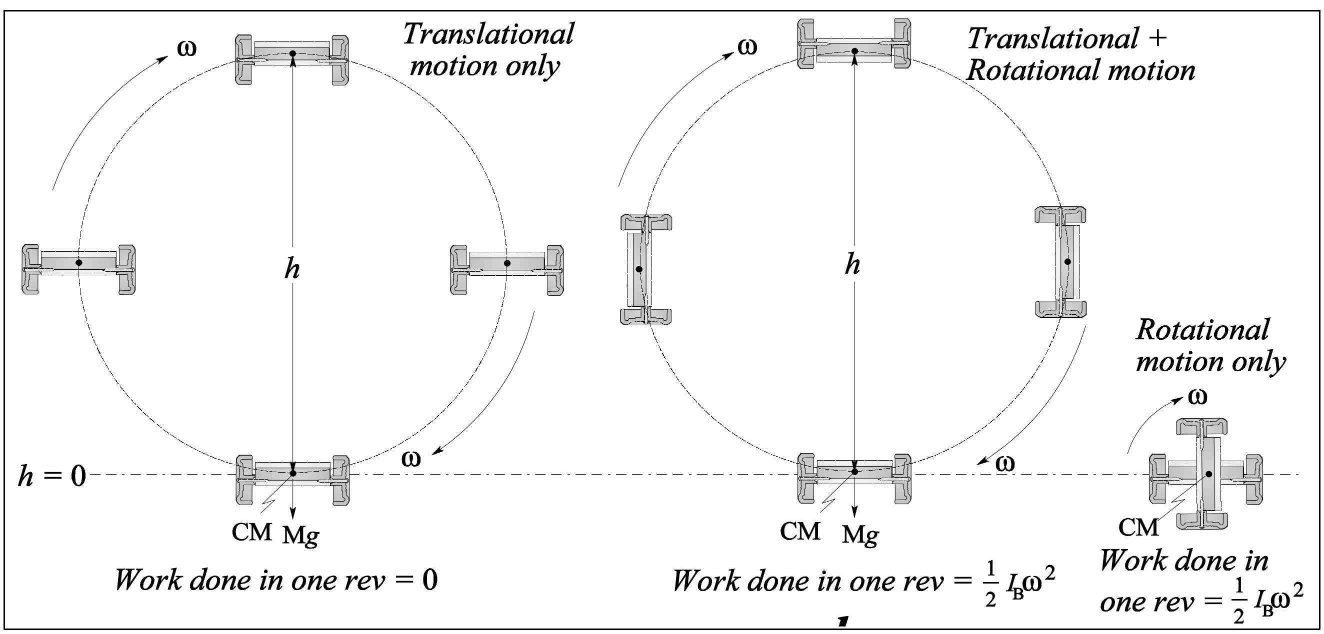

Figure 1 - *The Standard Car (wide body) of* **Figure 2** *on a rotating hoop.*

In **Figure 1** we are looking at 4 positions of the rear view of a PWD car that is being rotated on a hoop of diameter h and negligible mass. The car has a frictionless rod inserted in a lengthwise hole front to back in the body through the center of mass (CM) position. The rod is attached to the hoop. In ***a***, as the hoop rotates, the car remains level, rather like the seats on a ferris wheel. No net energy is expended, meaning no net work is done, as the car is lifted to height h and then returned to its initial $h = 0$ starting position. In the ***b*** scenario, the car is glued to the lengthwise CM rod and as the hoop rotates 360° the car has made one complete revolution about its CM axis as shown in the lower right of the figure. Although the initial and final configurations in ***a*** and ***b*** are exactly the same, rotational energy $E_R = \frac{1}{2} I_B \omega^2$ has been expended in case ***b***.

Figure 1 thus illustrates these two very fundamental concepts in rigid body motion:

1) The motion of any rigid body may be resolved into two independent motions. These are the translational motion of the CM and the rotational motion around the CM. Sometimes this is referred to as "motion of and about the CM." Translational motion is that where every point in the rigid body follows parallel trajectories, either straight or curved.

2) The change in kinetic energy of a point mass in a gravitational field, without air drag or other frictional effects, is completely determined by the initial and final positions of the mass. The CM qualifies as a point mass. For any motion in a gravitational field, if the CM ends up at the same height as it started, no matter what path was travelled in between, there is no net change in either its kinetic or potential energy. What happens is that there is a free exchange between potential and kinetic energy at the "in between" positions such that their sum remains always constant.

Problem Setup

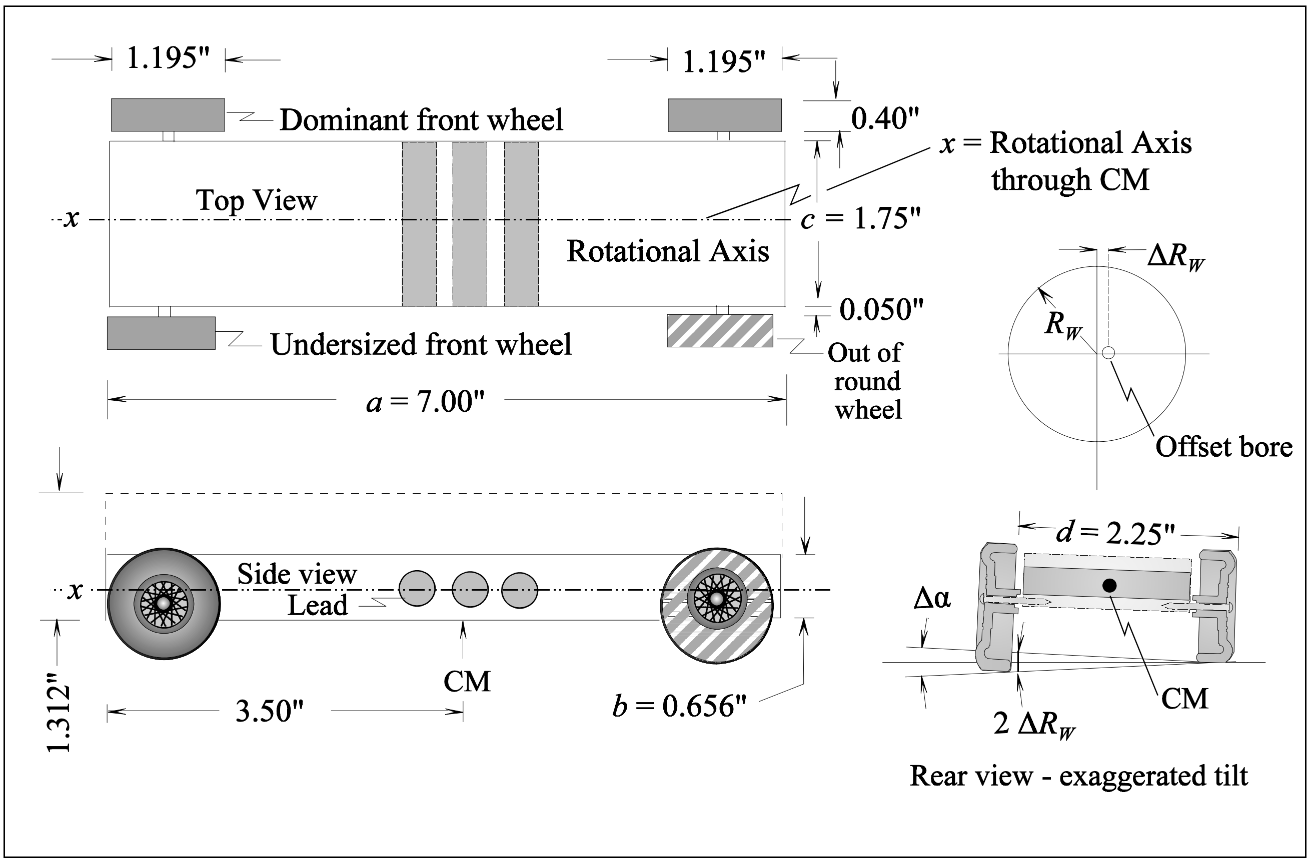

Figure 2 - *The Standard Car with a left rear wheel bump problem. The Rear View off-center tilt is greatly exaggerated*

Figure 2 shows what is defined as a Standard Car made by simply cutting the Cub's kit block in half, adding weight just behind the center, and attaching 4 wheels such that the left front does not touch the track. We will first write the equations that govern the car's dynamics and then examine speed losses from two problems with the left rear wheel. One problem is an out-of-round left rear wheel and another is a "bump" over which a perfect left rear wheel passes. The results do not depend on car wheel base. If there is a "bump" or an off-center rear wheel on the same side as the dominant front wheel, the body, being rigid, will still tilt approximately the same.

Note that the CM is slightly towards the rear, such rear weighting being common for most cars. In the rear weighting case a front wheel being off-center will not tilt the whole body to one side, but rather will raise/lower the whole front of the car. This will cause some rotation of the car body around an axis through the CM and perpendicular to the side of the body. The rotation angle will however be considerable less than the side tilt rotation angle shown at the lower right of **Figure 2**. Therefore we will neglect the much smaller front wheel rotation on the body of a rear weighted car.

Energy Laws

The approach to the problem will use the conservation of energy laws. The total energy, due to a certain starting ramp height $y = h$ of the center of mass of the car above the finish line level, is all potential and is given by

$$E_P = Mgh \tag{1}$$

Here M is the mass of the car, g is the acceleration of gravity, and E_P is the potential energy. After the start, this potential energy is all converted into kinetic energy E_K on the straight level run to the finish, thus at the finish line where $y = 0$ we have

$$E_K = \frac{1}{2}Mv^2 \tag{2}$$

Actually, one could look at it this way. Anywhere in a gravitational field, we have the total car energy E_T as

$$E_T = E_P + E_K = Mgy + \frac{1}{2}Mv^2 = constant \tag{3}$$

So when $v = 0$ at the starting CM height $y = h$ we have

$$E_T = E_P = Mgh = constant \tag{4}$$

And when we have the height y unchanging at a reference value $y = 0$ we have

$$E_T = E_K = \frac{1}{2}Mv^2 = constant \quad \text{(Note all these } \textit{constants} \text{ have the same value)} \tag{5}$$

Thus, since two quantities that equal the same quantity must equal each other, the energy at the start must equal the energy at the finish (wheel/axle friction, air drag, and wheel moment of inertia are assumed negligible). Therefore

$$\frac{1}{2}Mv^2 = Mgh \tag{6}$$

$$v = \sqrt{2gh} \tag{7}$$

This rather simple equation is very useful for determining race car velocity.

All the above energy is translational (because we neglect the small wheel rotational energy). However, if a wheel is out-of-round, the body can be twisted around an axis parallel to the direction of travel. For example, in **Figure 2**we have a wheel of radius R_W wherein the bore is off center by a small amount ΔR_W. In the lower right we see that the wheel can rotate the body by some angle α as the car rolls down the track. This rotational energy E_R is given by

$$E_R = \frac{1}{2}I_B\omega^2 \tag{8}$$

In (8), I_B is the moment of inertia of the body around a longitudinal axis through its CM and ω is the angular velocity of the rotation.

The approach here is to calculate (8) and use it to reduce the amount of energy (5) so that the overall energy remains constant. But as we can see from (6) and (7), this means a lower v. This will be calculated later.

First, we will look at the dynamics of the wheel bumps so we can deduce the angular velocity ω. In (9) below, Δα is the maximum angle change and Δt is the time corresponding to that angle change

$$\omega = \frac{\Delta\alpha}{\Delta t} \tag{9}$$

Case 1(a) - Out-of -Round (off - Center) Wheel

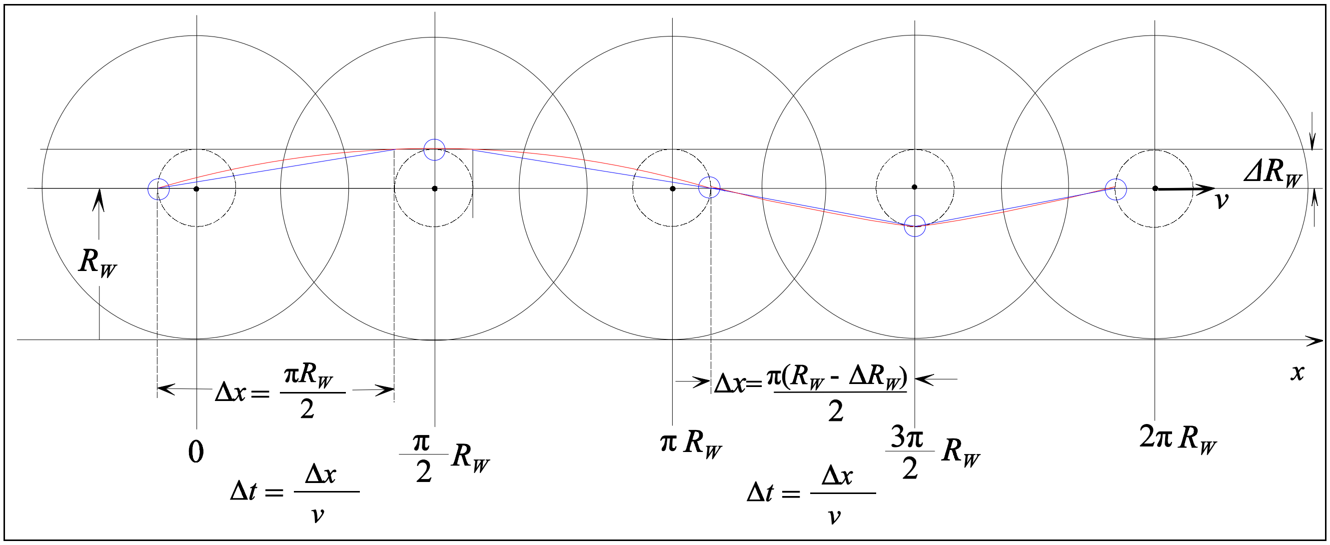

Figure 3 - *The motion of a point off center on a rolling wheel describes a trochoid (red line).*

Figure 3 shows a wheel rolling to the right at velocity ***v***. The bore hole is offset an amount ΔR_W. As the wheel rolls the bore hole describes the trochoid curve shown in red. It is similar to the cycloid curve formed by a point on the rim of the rolling wheel but it is a much shallower curve. Here the ΔR_W offset is exaggerated to 30% of the radius for clarity. We can approximate the red line at the left by the blue straight line over the distances shown. The red line curvature does not change much at the very top of the trochoid so the blue line only extends over 1/4 rotation. As one can see from the figure, the rolling distance (and the time Δt) is slightly longer when the wheel bore is raising the near body side compared to when is dropping it below its level position. The total rolling distance when either increasing or when decreasing the angle α is

$$\Delta x = \frac{\pi}{2}(2R_W - \Delta R_W) \tag{10}$$

However, even a large ΔR_W could be like 0.010" and the wheel diameter 2 R_W is about 1.20". So the effect on the Δα we will consider will usually be less than 1%. Thus, we can neglect ΔR_W in (10) and get for a rolling distance and time for an α increase (or a decrease),

$$\Delta t = \frac{\Delta x}{v} = \frac{\pi R_W}{v} \tag{11}$$

The angle change in α for this Δt is Δ α, and is, in radian measure (2π radians = 360º), to a good approximation given by its tangent, which is simply the "rise over run". The run, as seen in **Figure 1**, is the distance d = 2.25" and the rise is of course $2\Delta R_W$. So we have from (9)

$$\omega = \frac{\Delta\alpha}{\Delta t} = \frac{2v\Delta R_W}{d\Delta x} = \frac{2v\Delta R_W}{d\pi R_W} \tag{12}$$

From (7) we can substitute for v giving

$$\omega = \frac{2\Delta R_W\sqrt{2gh}}{d\pi R_W} \tag{13}$$

The rotational energy from one "bump" is thus from (8)

$$E_R = \tfrac{1}{2} I_B \omega^2 = I_B g h \left(\frac{2\Delta R_W}{d\pi R_W} \right)^2 \tag{14}$$

The moment of inertia of the rectangular block of sides *a*,*b*,*c*, of mass *m*, when being rotated around its longitudinal CM axis is shown in the index of many engineering texts as

$$I_B = \tfrac{1}{12} m (b^2 + c^2) \tag{15}$$

The final expression for the rotational energy for one “bump” is therefore, from (14)

$$E_R = \tfrac{1}{12} m (b^2 + c^2) g h \left(\frac{2\Delta R_W}{d\pi R_W} \right)^2 \tag{16}$$

Now let us consider just the straight horizontal run section. We can see from **Figure 2** that every time the wheel rotates once there is both a rotation of the body mass up above horizontal followed by a rotation of the body mass below horizontal Don’t worry about gravity effects on the CM if it moves up and down some during the bump—just like explained in **Figure 1** this intermediate motion does not use energy. Suppose the coasting distance is length *l*. Then the wheel does *N* rotations where *N* is given by the coasting length divided by the wheel circumference.

$$N = \frac{l}{2\pi R_W} \tag{17}$$

So for the 2*N* “twists” in distance *l* we have an associated kinetic energy of body rotation as

$$E_R = \frac{2N}{12} m (b^2 + c^2) g h \left(\frac{2\Delta R_W}{d\pi R_W} \right)^2 \tag{18}$$

The total energy at the finish is now still equal to the total starting energy *Mgh* so that.

$$E_R + \tfrac{1}{2} M v_2^2 = M g h \tag{19}$$

The new slower velocity v_2 at the finish and resulting time t_2 can be obtained from (19) as

$$v_2 = \sqrt{2gh - \frac{2E_R}{M}} \tag{20}$$

$$t_2 = \frac{l}{\sqrt{2gh - \frac{2E_R}{M}}} \tag{21}$$

$$t = \frac{l}{\sqrt{2gh}} \quad \text{where } t \text{ is the time with no wheel bumping, i.e., } E_R = 0 \tag{22}$$

Using (18) in (20) and setting up a spreadsheet solution for the times, one can get the time difference $t - t_2$ at the finish line as the out-of-round offset is ΔR_W varied. The time loss can be converted to an equivalent distance at the finish by multiplying by *v*. A graph will be presented later after we consider next a discrete bump on the track.

Case 2 - Normal Well-Centered Round Wheel Rolls Over Obstacle

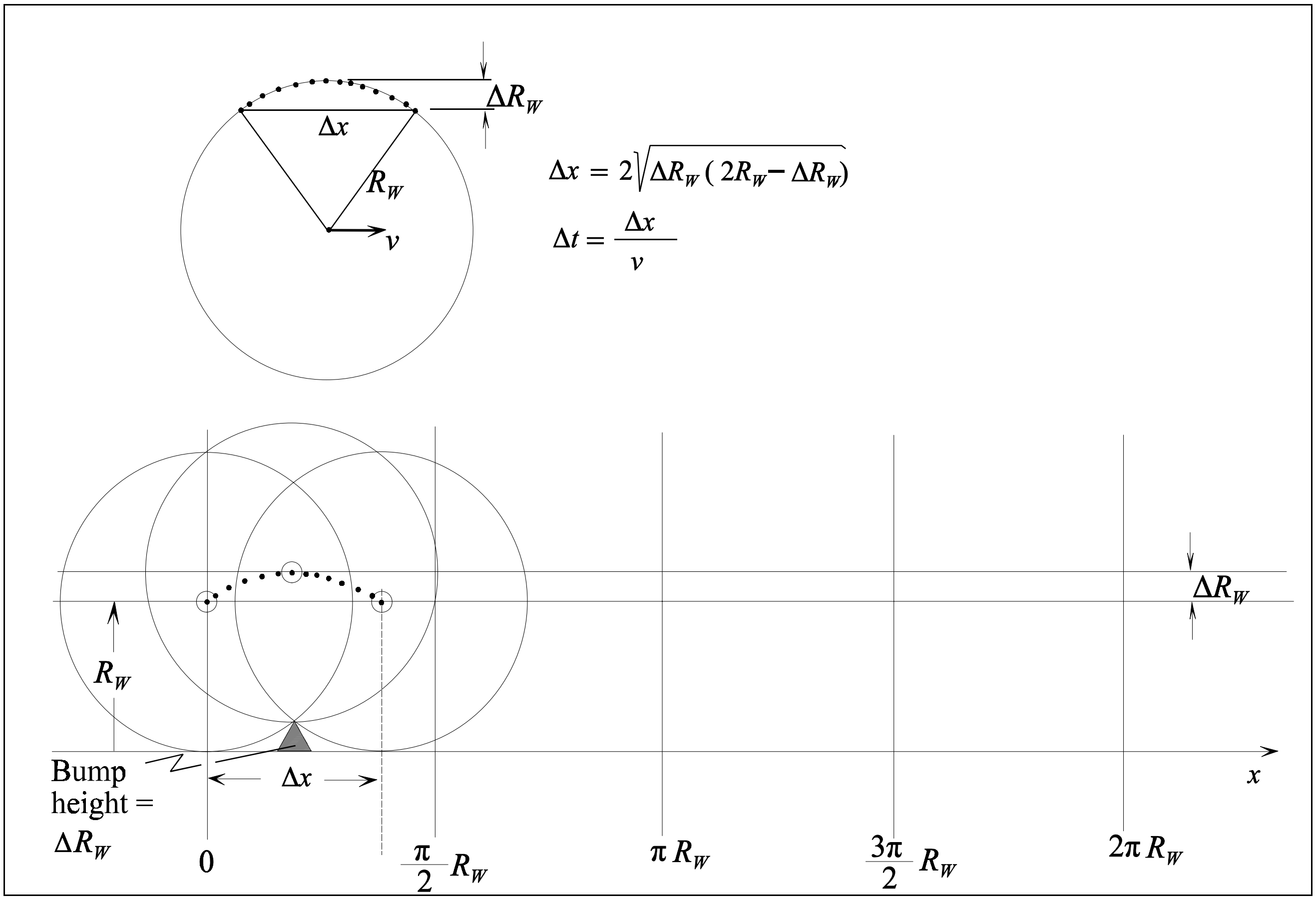

Figure 4 - *The motion of the axle caused by a sharp bump on a rolling wheel.*

In **Figure 3** we show the case where a normal wheel twists the body by rolling over a bump. In this case the axle/bore hole traces a circular arc segment of height R_W as a deviation from an otherwise straight trajectory. Again, we get the time for the deflection from Δx, the distance traveled during the deflection. The formula for the length Δx may be found at the math world website http://mathworld.wolfram.com/CircularSegment.html . It is

$$\Delta x = 2\sqrt{\Delta R_W(2R_W - \Delta R_W)} \tag{23}$$

The angular velocity may be calculated similar to the last case as

$$\omega = \frac{\Delta\alpha}{\Delta t} = \frac{v\Delta R_W}{d\Delta x} = \frac{v\Delta R_W}{2d\sqrt{\Delta R_W(2R_W - \Delta R_W)}} \tag{22}$$

The total body rotational energy may be obtained as before from (16). Of course the number of bumps, (later we will use N_B to denote this number), must be estimated from the track surface condition. An uncleaned track might have for example 20 transverse brush bristles, each of typical diameter 0.005".

The time difference between a clean and uncleaned track may then be found similar to equations (18) through (23).

Parameter Table

Table 1 shows parameter values used in the preceding formulas to calculate finish differences in fractions of an inch as a function of ΔR_W.

The calculation results are for a 32-ft track which has a 16 ft horizontal run with the finish line 2 ft from the end. This gives a 14 ft coasting distance.

The ramp height to the center of the car (which is also the CM for the cars shown) is a fairly typical 47 inches.

The wheels weigh about 10 grams for the 3 touching and the raised front wheel is counted as part of the body mass of 131.75 g.

Notice that even with weights concentrated at the body center, the moment of inertia for twisting the body around a long edge is not affected as long as the weights uniformly traverse the whole width.

Table 1 - *Parameters used in various calculations*

Parameter	Symbol	Value (Eng)	Units (Eng)	Value (cgs)	Units (cgs)
Body length	a	7.00	in	17.78	cm
Body height	b	0.656	in	1.67	cm
Body width	c	1.75 & 1.00	in	4.45 & 2.54	cm
Ramp height to CM	h	47.00	in	119.38	cm
Tilt distance hypotenuse	d	2.25	in	5.715	cm
Wheel radius	R_W	0.5975	in	1.518	cm
Offset or bump height	ΔR_W	varies	in	varies	cm
Horizontal run length	l	14	ft	426.72	cm
Coast velocity (no friction or wheel bumps)	v	11.9	mph	483.72	cm/s
Car mass	M	5.00	oz	141.75	g
Body mass	m	4.66	oz	131.75	g
Body moment of inertia (For c = 1.75 & 1.00")	I_B	-	-	248 & 101	g cm^2

Example of a Narrow Body

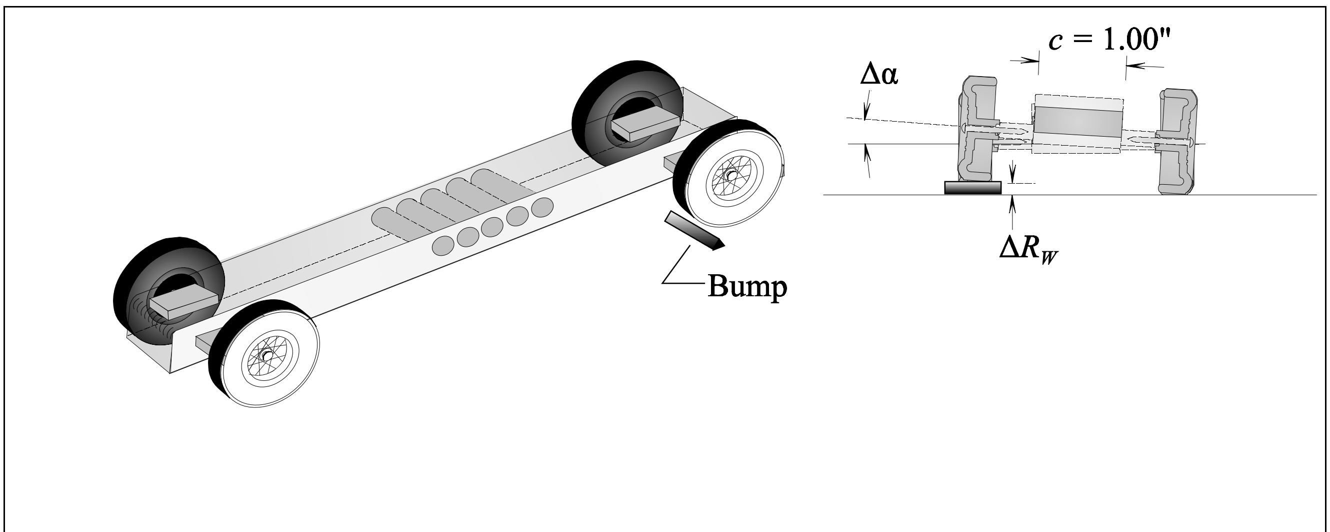

Figure 5 - *This car is identical to the* ***Fig. 2*** *car except here the body is only 1.00" wide with more but shorter pieces of lead worm. Also the wheels/axles are supported on light but strong wood struts (eg. basswood) with the same spacing as before.*

Calculation Results

Figure 6 shows the results for Case 1 . **Lecture 22** will present measurements on out-of-round amounts for stock Cub Scout wheels. They range from 0.003" to 0.013". In the former case the difference at the finish line is only a few thousandths of an inch which would not show up on a timer. Full scale (0.25") on the graph is only about 0.0012 seconds. It should be mentioned that these effects are for the coasting run only, and a similar effect on the ramp is estimated to add another 25% to finish line difference. These results are for a shorter 14 ft coast typical of a 32 ft track, so scale up the finish line distances proportionally for longer horizontal runs.

Figure 6 - CASE 1 - OUT OF ROUND REAR WHEEL

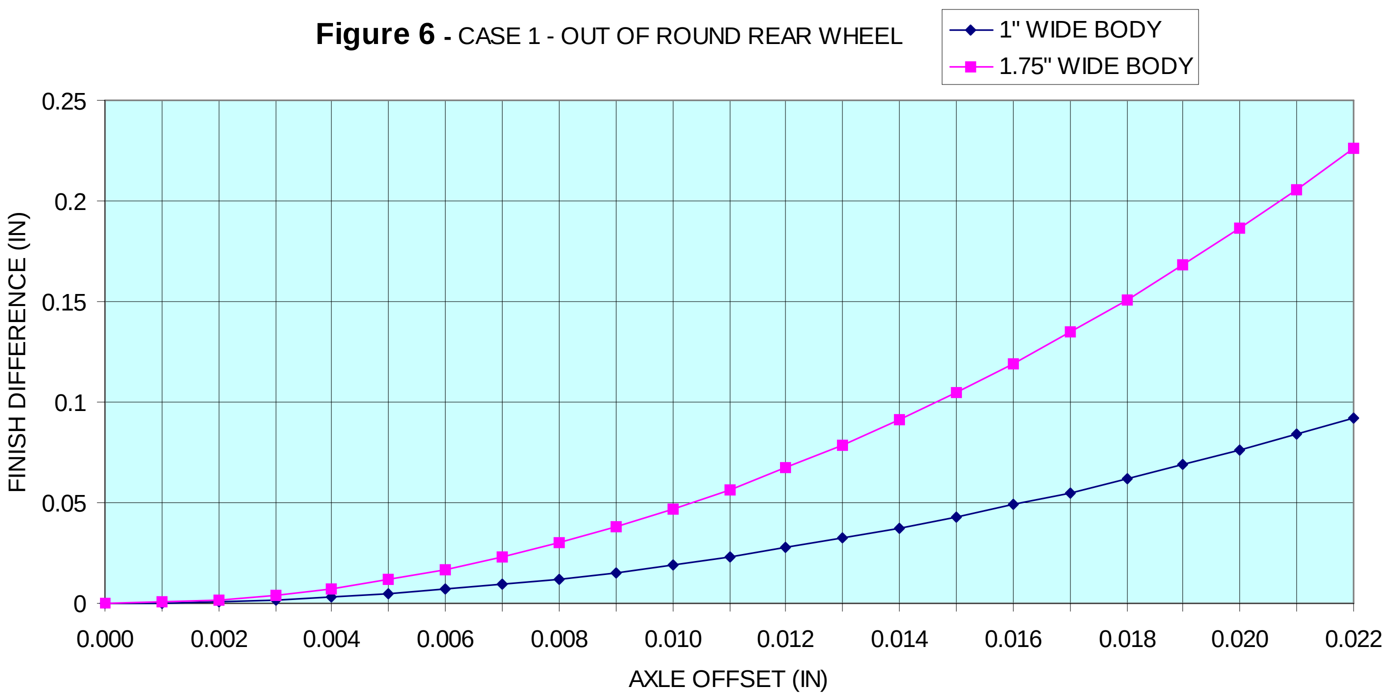

Figure 7 shows what it would cost at the finish line for a dirty track sprinkled with up to 22 crosswise brush bristles or other roughness that would cause up to 22 rear wheel bumpings. Small 0.002" high bumps are not too much of a problem, but with a wide body and 0.005" high bumps, 20 bumps could cost you about two-tenths of an inch at the finish line. Note an unsanded tread mold mark 0.005" high could give 45 bumps on the 14 ft coast and cost about ½" at the finish.

Figure 7 - CASE 2 - SHARP BUMP